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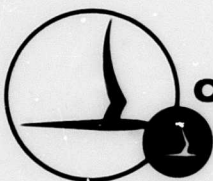
CONTRAILS FORECASTING

Prepared for:

OFFICE OF NAVAL RESEARCH AND THE
BUREAU OF NAVAL WEAPONS
DEPARTMENT OF THE NAVY
WASHINGTON 25, D.C.

By: James E. Jiusto and Roland J. Pilié
CAL Report No. VC-1660-P-3
Contract No. Nonr-3672(00)
August 1964

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CONTRACT NO. Nonr-3672(00)

AUGUST 1964

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ABSTRACT

The principles of contrail formation and dissipation are reviewed. Methods are described for predicting the occurrence of contrails and for estimating their persistence in a form suitable for weather station use. Contrail "climatology" for long-range forecasts is discussed as are factors that affect the visual detection of contrails.

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LIST OF SYMBOLS

c_p	specific heat of air at constant pressure
H	heat of combustion of fuel
p	atmospheric pressure
P, P'	arbitrary points on contrail diagrams representing ambient conditions (p, T, RH)
RH	relative humidity
T	ambient temperature
T_c	warmest (critical) temperature at a given pressure that will foster contrail formation
T_d	temperature at and below which contrails can form in a perfectly dry atmosphere (0% RH)
T_i	temperature above which contrails that form <u>must</u> be super-saturated with respect to ice
w	mixing ratio of air
w_s	saturation mixing ratio with respect to water
w_i	saturation mixing ratio with respect to ice
W	mass of water vapor produced per mass of fuel burned
$\Delta w / \Delta T$	ratio of water vapor to heat produced in fuel combustion
$\Delta \omega_L$	maximum concentration (density) of condensed water in contrail - g (water)/kg (air)
θ	scattering angle subtended at contrail by observer and sun's rays

1. INTRODUCTION

Condensation trails can significantly decrease the effectiveness of operational aircraft in combat situations. The enemy's ability to detect aircraft is greatly enhanced by the presence of these tell-tale cloud streamers. Interception by enemy aircraft, missiles, or ground fire is thereby facilitated.

To diminish aircraft detection during missions, one must predict the altitudes at which contrail formation is probable so that these flight altitudes can be avoided. Long-range (time and distance) military planning can be enhanced by knowledge of the synoptic, geographic and seasonal factors influencing contrail formation. Such contrail prediction is the function of the meteorologist, who must provide the flight briefing and weather data for use in mission planning. To most effectively provide this information, the meteorologist should:

1. Understand some of the basic principles of contrail formation and dissipation (Sections 2.1, 4.1)
2. Be able to prepare systematic forecasts of contrail occurrence for areas where atmospheric parameters are reasonably well known (Sections 2.2-2.6)
3. Know how to analyze synoptic and climatological data for long range estimates of contrail occurrence (Section 5)
4. Be able to estimate contrail persistence (Section 4)
5. Appreciate some of the limitations of the contrail forecasting techniques (Section 3).

The contrails forecast manual which this document replaces emphasized only item 2 above - contrail prediction based on known atmospheric conditions. The present manual enlarges on this important function and attempts, as well, to supply the meteorologist with techniques and information to fulfill the other listed objectives. The reader may choose to emphasize those specific sections of the manual, indicated in parentheses, that relate most directly to his particular mission assignment.

2. FORECASTING CONTRAIL FORMATION

2.1 Principles of Contrail Formation

Meteorologists recognize that contrails are long narrow clouds of water droplets and/or ice crystals that frequently form behind aircraft. Water vapor liberated in the fuel combustion process can, under appropriate ambient temperature conditions, saturate the air in the exhaust wake and cause condensation. For normal cruising altitudes of jet aircraft, cold regions conducive to contrail formation are frequently encountered.

In addition to these exhaust trails, there is a condensation phenomenon associated with the pressure reduction and resultant adiabatic cooling of air flowing around propeller tips and wing tips. These so-called aerodynamic trails, being transient and comparatively rare at operational altitudes, are not considered an operational problem. Similarly, dissipation trails or distrails, which are clear tracks produced in existing thin clouds by the heat of fuel combustion released in the aircraft wake, are rare and of minor consequence.

The theory of exhaust contrail formation was first formulated by Appleman (1953) [1]* and substantially verified with some qualifications by laboratory and atmospheric observations. His development assumes that all contrails freeze and that a contrail will be visible if it possesses a water concentration of 0.004 g/m^3 . (In Section 3, the validity of these assumptions is discussed.)

When an aircraft passes through the atmosphere, the engine produces a large quantity of water vapor and heat as a result of fuel combustion in the engine. The exhaust gases undergo mixing with the ambient air. The water vapor introduced tends to increase the ambient relative humidity and bring the air closer to saturation, whereas the heat released increases air temperature and thus tends to decrease the relative humidity. The formation of a contrail depends, therefore, upon the ratio of water and heat produced by fuel combustion, the wake or entrainment characteristics of the aircraft, and the original temperature and humidity conditions of the undisturbed air.

*Numbers in brackets denote references which are listed in Section 7.

The fuel used in airplanes (propeller and jet) is a hydrocarbon which produces approximately 1.3 grams of water and 10,400 calories of heat for every gram of fuel consumed [2]. The ratio of these values is relatively constant for different aircraft power settings and approximately the same for all aviation fuels in use. This proportionality constant expressing the change in mixing ratio Δw to the change in temperature ΔT in the aircraft wake* is given by

$$\Delta w / \Delta T = 1000 W c_p / H = 0.0295 \text{ g/kg}^\circ\text{C}$$

where W is the mass of water vapor per gram of fuel burned
 H is the heat of combustion of the fuel and
 c_p is the specific heat of air at constant pressure.

In the case of jet aircraft, all of the heat and moisture produced in fuel combustion is injected into the wake. For piston engine aircraft, on the other hand, although all the water produced is added to the wake, a significant portion of the heat is dissipated outside the wake in the form of radiant heat losses from the engine and mechanical losses from the propeller. The exact amount of heat lost in this manner varies with the type of aircraft, these losses averaging about 20 to 40 percent of the total. Since the heat contribution to the wake tends to decrease the probability of contrail formation, propeller-driven aircraft will produce contrails more readily than jets under the same atmospheric conditions. Jet aircraft operation is, of course, of primary military importance.

The amount of air entrained into the exhaust wake varies continuously from near zero immediately behind the aircraft to an extremely large amount far behind it. The rate at which air is entrained varies with the type of aircraft, power setting, speed and with the density and stability of the atmosphere. The ratio of entrained air to exhaust gas, at a given distance behind the aircraft, is greatest when the aircraft is operating at its most efficient speed and altitude. In all cases, irrespective of aircraft type or operation, this ratio varies from zero close to the aircraft to a large value approaching infinity some distance back.

* The aircraft wake is defined as that portion of the atmosphere that is thermodynamically influenced during aircraft passage.

Since for jet aircraft the amount of heat and moisture added to the wake per gram of fuel consumed is substantially constant and since complete mixing occurs, the occurrence or nonoccurrence of contrails depends only upon the temperature, pressure and relative humidity of the undisturbed air. In general at any pressure level in the atmosphere, the colder the air temperature and more humid the air, the greater will be the tendency for contrails to form.

2.2 Graphical Prediction Technique

The expectation of contrail formation by jet aircraft is indicated in Figure 2.1 as a function of the three controlling atmospheric parameters - pressure, temperature and relative humidity (p, T, and RH). On this chart the meteorologist should locate the point or points representing existing or predicted pressure and temperature conditions along the flight route. If the p-T point lies to the right of the 100 percent RH curve, contrails will never form irrespective of ambient relative humidity. If the p-T point lies to the left of the 0 percent RH curve, contrails will always form irrespective of ambient humidity. If the point lies between the 0 and 100 percent RH curves, contrail formation will be dependent on ambient relative humidity equalling or exceeding the indicated RH value. For example, an aircraft being flown at an altitude corresponding to a pressure of 200 mb and a temperature of -50°C will produce contrails only if ambient relative humidity exceeds about 92 percent.

The dashed line in the figure represents the International Civil Aeronautical Organization (ICAO) standard atmosphere. Depending on geographic location and season, arctic or tropical standard atmospheres may be more appropriate.

Numerical values of critical contrail formation temperature for jet aircraft are listed in Table 2.1. Similarly, Figure 2.2 and Table 2.2 indicate critical contrail formation conditions for propeller driven aircraft [3].

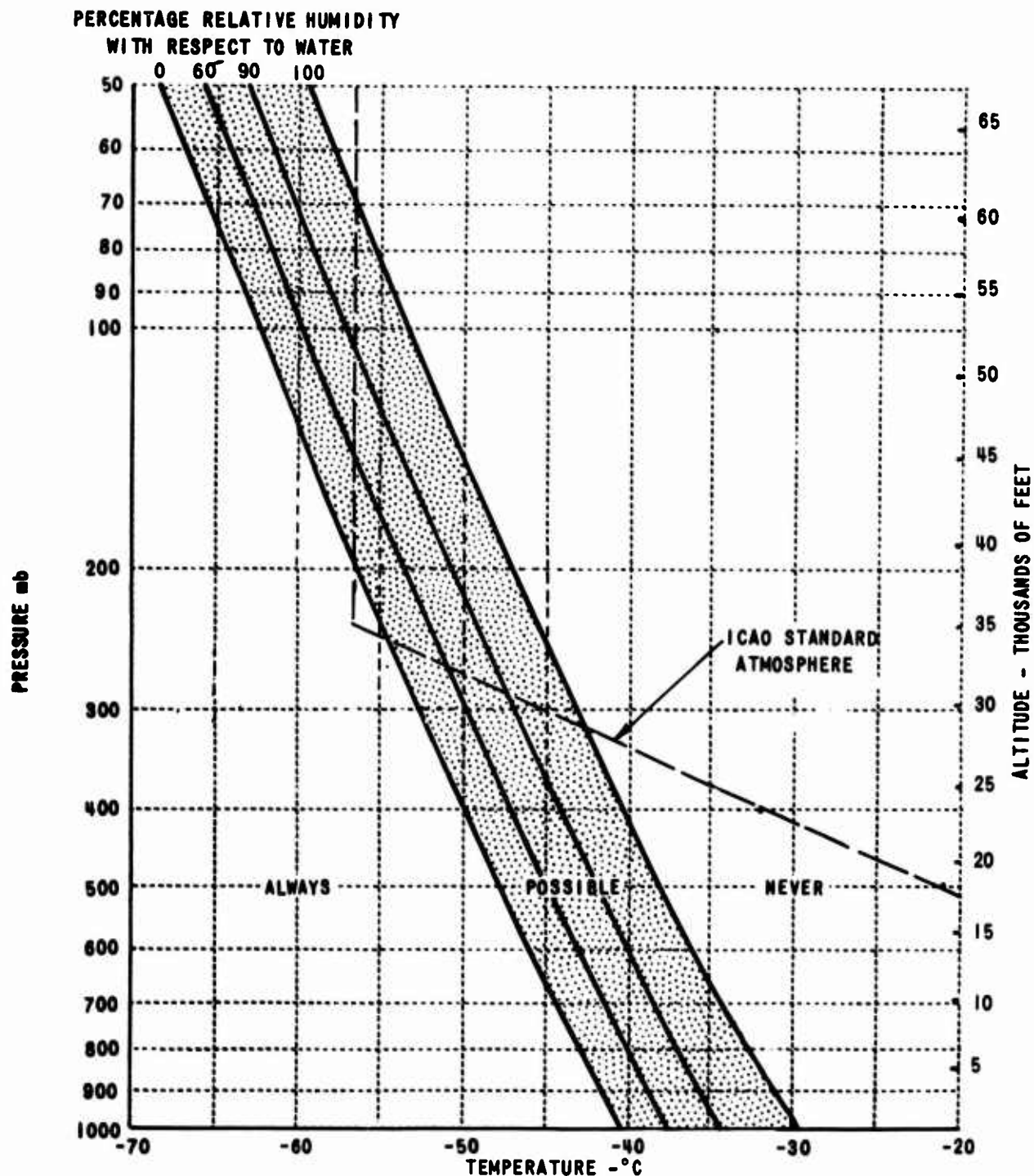


Figure 2.1 JET AIRCRAFT CONTRAIL FORMATION AS A FUNCTION OF AMBIENT PRESSURE, TEMPERATURE AND RELATIVE HUMIDITY

In the unshaded region to the left of the 0% RH line, theory predicts that contrails will always occur; to the right of the 100% RH line contrails should never occur; and in the shaded region contrails are possible depending on ambient humidity.

Table 2.1

Contrail Formation Temperature - Negative °C

$$\Delta w / \Delta T = 0.0295 \text{ g/kg}^\circ\text{C}$$

Pressure (mb)	Relative Humidity with Respect to Water - Percent					
	0 (Td)	60	90	100 (Td)	0 *	100 *
50	68.1	65.8	63.4	59.4	(69.8)	(65.0)
100	62.4	60.0	57.4	53.7	(63.3)	(58.3)
150	59.0	56.4	53.8	50.1	(59.6)	(54.0)
200	56.4	53.8	51.0	47.5	(56.8)	(50.4)
300	52.6	50.0	47.2	43.7	(53.0)	(45.9)
400	50.0	47.2	44.2	40.5	(50.2)	(42.6)
500	47.8	45.0	42.0	38.2	(48.0)	(39.9)
600	46.0	43.1	40.0	36.1	(46.2)	(38.0)
700	44.5	41.5	38.3	34.3	(44.7)	(36.2)
800	43.2	40.2	36.9	32.7	(43.3)	(34.4)
900	41.9	38.9	35.6	31.2	(42.0)	(32.9)
1000	40.6	37.7	34.5	29.9	(49.7)	(31.5)

*() $T_{c,d}$ Value Allowing for the Production of 0.004 g/m^3 of Condensate to Make a Nonfreezing Contrail Visible

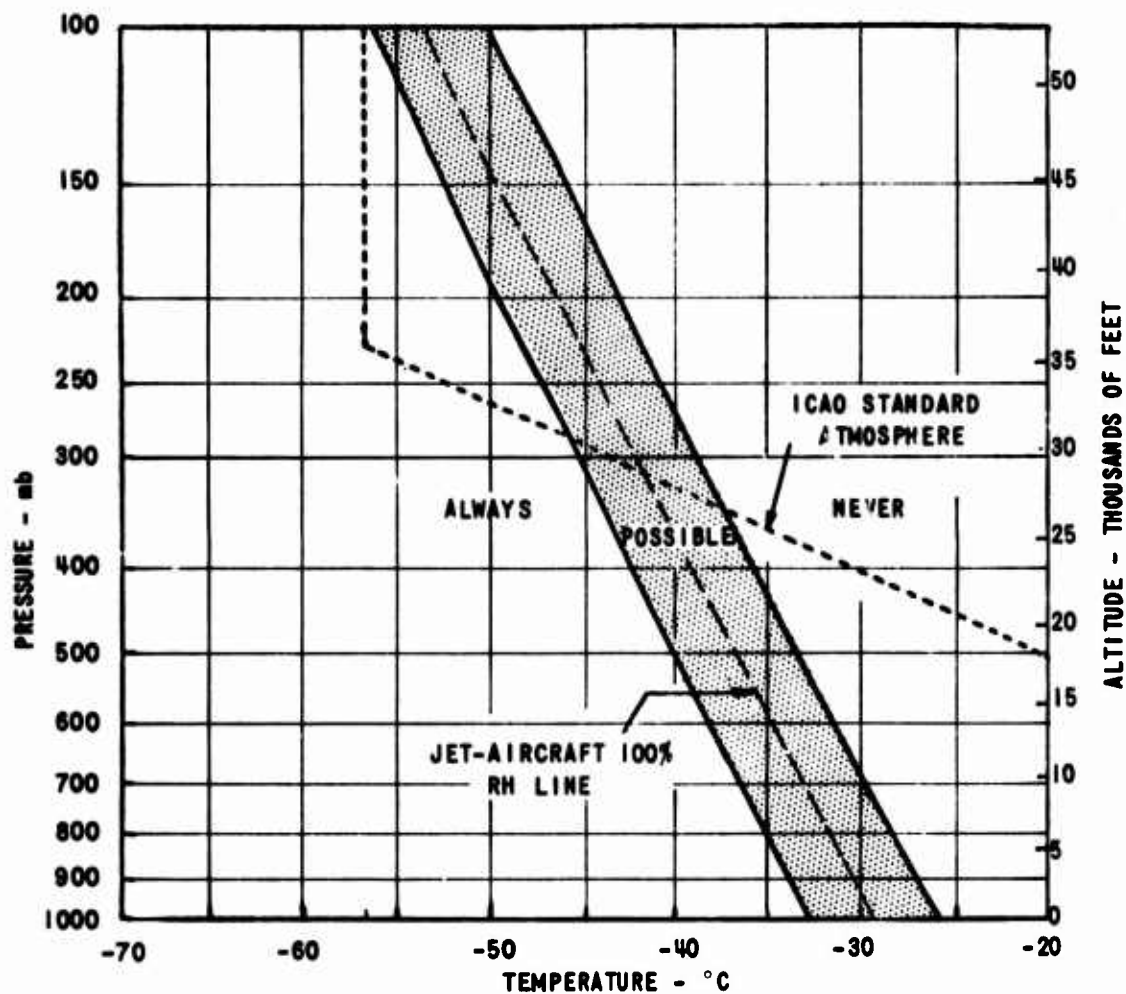


Figure 2.2 PISTON-AIRCRAFT CONTRAIL-FORMATION GRAPH

The interpretation of this graph is identical to that of Figure 2.1 for jet aircraft. The 100% RH line from the jet aircraft graph is superimposed to show that piston aircraft can produce contrails at warmer temperatures than jet aircraft

Table 2.2
Critical Temperatures for Contrail Formation
by Propeller Aircraft

Pressure (mb)	Temperature - Negative °C	
	Left Boundary	Right Boundary
100	56.0	50.0
150	52.3	46.0
200	49.6	43.2
250	47.4	41.0
300	45.6	39.0
400	42.7	36.0
500	40.3	33.7
600	38.4	31.6
700	36.8	29.9
800	35.2	28.3
900	34.0	27.0
1000	32.9	25.9

2.3 Graphical Aids

The meteorologist will find it helpful to prepare one plastic overlay for contrail forecasting for jet aircraft and one for propeller-driven aircraft. This can be done in the following manner:

1. On an adiabatic chart typical of the type normally used for plotting upper air soundings, plot the critical contrail formation points of Table 2.1 and draw the curves using Figure 2.1 as a guide.

2. Trace these lines on a transparent plastic sheet together with the -40°C isotherm and the 1000 mb isobar as reference marks to permit subsequent correct placing of the overlay on adiabatic charts. Label the overlay "Contrail Formation for Jet Aircraft."

3. Repeat the procedures outlined in 1 and 2 above using Table 2.2 and Figure 2.2 as guides. Label this overlay "Contrail Formation for Propeller-driven Aircraft."

2.4 Procedure for Contrail Prediction at a Given Station

Plot the temperature profile for the station of interest on an adiabatic chart and record the numerical values of ambient relative humidity along the profile.

1. If the humidity measurements are available from the current raob, these should be used.

2. If humidity measurements are not available, as is generally the case, the following criteria may be used:

- a. In the complete absence of data, a relative humidity of 40 percent with respect to water in the upper troposphere is a reasonable value. In the stratosphere humidity is generally low, and an assumed value of 0 percent may be used with reasonable accuracy.

- b. If the upper tropospheric air flow is known, use 60 percent relative humidity for flow from a moist region and 0 percent for flow from a dry region.

c. If cirrus clouds are present, use 60 percent relative humidity for the level of the cirrus clouds.

After approximating relative humidity in the above manner, the meteorologist should place the appropriate plastic overlay on the current adiabatic chart and note whether the sounding intersects the array of lines on the overlay. If the sounding is to the right of the appropriate relative humidity line (for jets), contrails will not form. If the sounding is to the left, contrails will form. If the relative humidity was approximated, a few degrees margin of error should be allowed both to the right and left of the relative humidity line. If the sounding falls within this "margin of error" region, possible contrail formation should be indicated.

2.5 Alternate Graphical Solution

A contrail prediction diagram, which provides identical results as the first method but contains additional information about contrail characteristics, is illustrated in Figure 2.3. The heavy curved line in the phase diagram represents the saturation mixing ratio relative to water versus temperature at 300 mb. The straight lines having $\Delta w/\Delta T$ slopes of $0.0295 \text{ g/kg}^\circ\text{C}$ indicate changing conditions of temperature and moisture in the aircraft exhaust wake as mixing with the environment progresses. The two extremities of such a straight line represent temperature and humidity conditions at the aircraft tailpipe and in the undisturbed environment respectively; intermediate wake conditions are represented by points along the straight line. Mixing will progress until the wake has reached ambient temperature and humidity.

Consider an aircraft flying through ambient air characterized by point P (-51°C , 80 percent RH) in Figure 2.3. The air-exhaust mixture, in approaching conditions of point P, exceeds water saturation, and hence a contrail will form. An indication of the maximum concentration of condensed liquid water $\Delta\omega_L$ in the exhaust wake is given by the degree to which water saturation is exceeded, i.e., the maximum separation distance between the water saturation curve and the straight line* depicting wake conditions.

*The line is actually slightly curved where it exceeds the saturation mixing ratio curve owing to the heat of condensation released as droplets form.

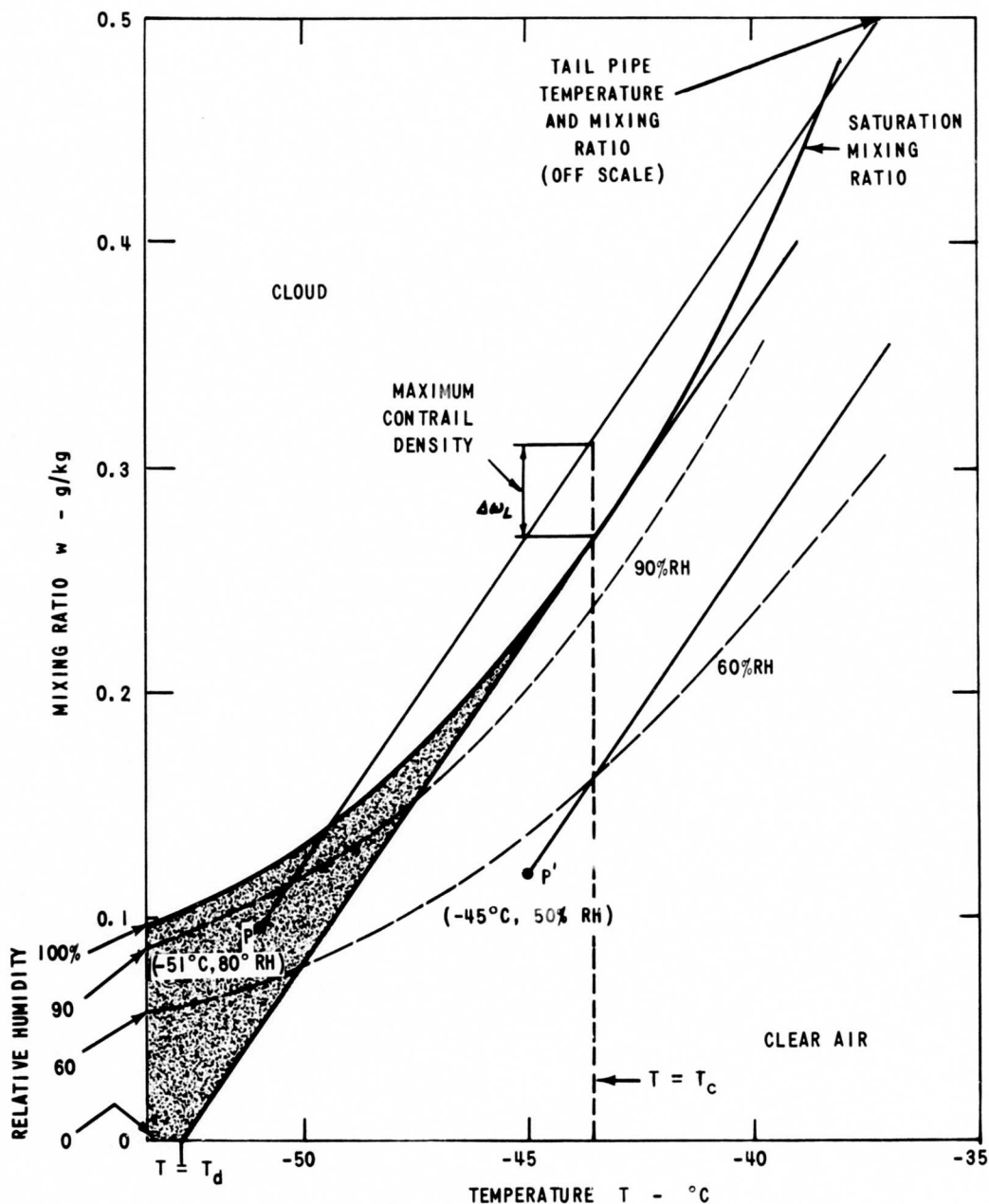


Figure 2.3 300 mb CONTRAIL PREDICTOR DIAGRAM

The curved line represents saturation mixing ratio versus temperature at 300 mb, while the straight lines with slope $\Delta w/\Delta T$ indicate aircraft wake conditions. The lower left end of the straight line represents ambient temperature and pressure conditions which the exhaust gases will approach far back in the aircraft wake. Atmospheric conditions in the shaded region, such as at point P, will give rise to contrail formation. T_c represents the warmest temperature (100% RH) that will foster contrails. Temperatures colder than T_d will promote contrails even in a perfectly dry atmosphere. Relative humidity curves of 0, 60, 90 and 100% are indicated respectively by the abscissa, two dashed curves and the saturation mixing ratio curve.

Now consider ambient conditions represented by point P' (-45°C, 50 percent RH): The straight line depicting aircraft wake conditions does not cross the saturation curve in reaching P'. Thus, the wake remains subsaturated with respect to water and contrails will not form.

Thus to predict whether contrails will form, the meteorologist has only to locate point P on the chart corresponding to existing ambient conditions (p, T, RH) at the desired flight level. From point P a line with slope $\Delta w/\Delta T$ is then drawn to see if it intersects the water saturation curve.

Threshold or critical conditions for contrail formation are represented by a line having a $\Delta w/\Delta T$ slope of 0.0295 and tangent to the saturation mixing ratio curve. In a saturated atmosphere, the warmest temperature that will permit contrail formation is T_c , the temperature corresponding to the point of tangency of the two curves. For a perfectly dry atmosphere, contrail occurrence would require temperatures colder than T_d , the temperature at which the tangent line crosses the zero-humidity axis (the abscissa). At temperatures between T_c and T_d , contrail formation is dependent on ambient relative humidity. For example, at -50°C the relative humidity must exceed 58 percent in order that a contrail may be produced. In short, the hatched area in Figure 2.3 describes environmental conditions that will foster contrail formation at 300 mb.

The family of saturation mixing ratio curves for various pressure levels is shown in Figure 2.4. It is suggested that the forecaster prepare a separate chart for each pressure level of interest. Simple construction of a plastic right triangle with $\Delta w/\Delta T$ slope of 0.0295 g/kg°C will allow ready use of each chart as described above. This type of graphical presentation is advantageous in that it:

1. Clearly illustrates certain cloud physics principles involved in contrail formation
2. Provides quantitative information on the maximum amount of liquid water comprising the contrail

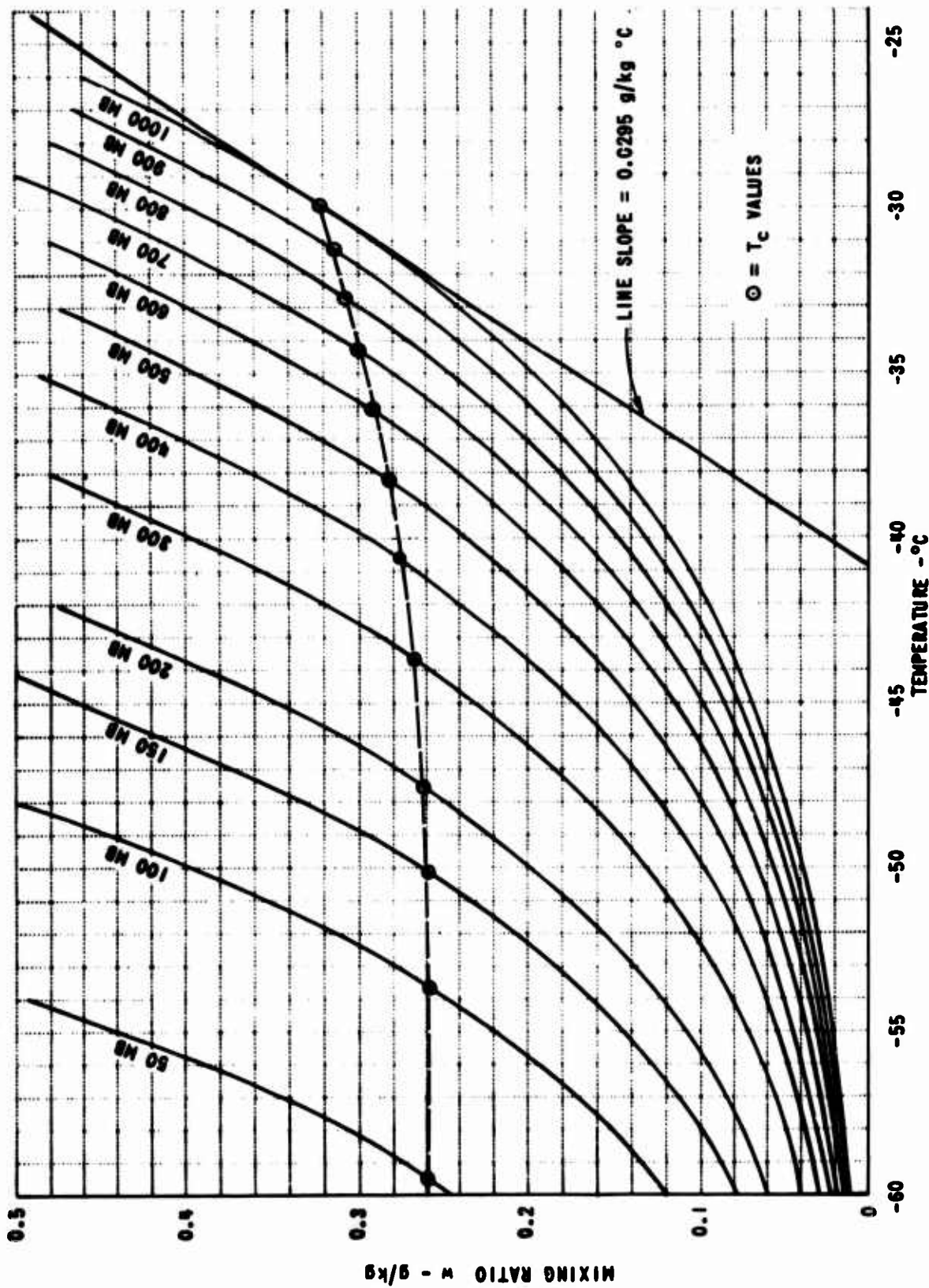


Figure 2.4 CURVES OF SATURATION MIXING RATIO OVER WATER vs TEMPERATURE AND PRESSURE

The warmest or critical temperature (T_c) at which contrails can form at a given pressure level is shown by a circled point. This is the point of tangency of the saturation mixing ratio curve and the straight line with slope of $0.0295 \text{ g/kg } ^\circ\text{C}$ ($\Delta w/\Delta T$).

3. Will accommodate any future fuel characteristics simply by altering the $\Delta w/\Delta T$ ratio, and

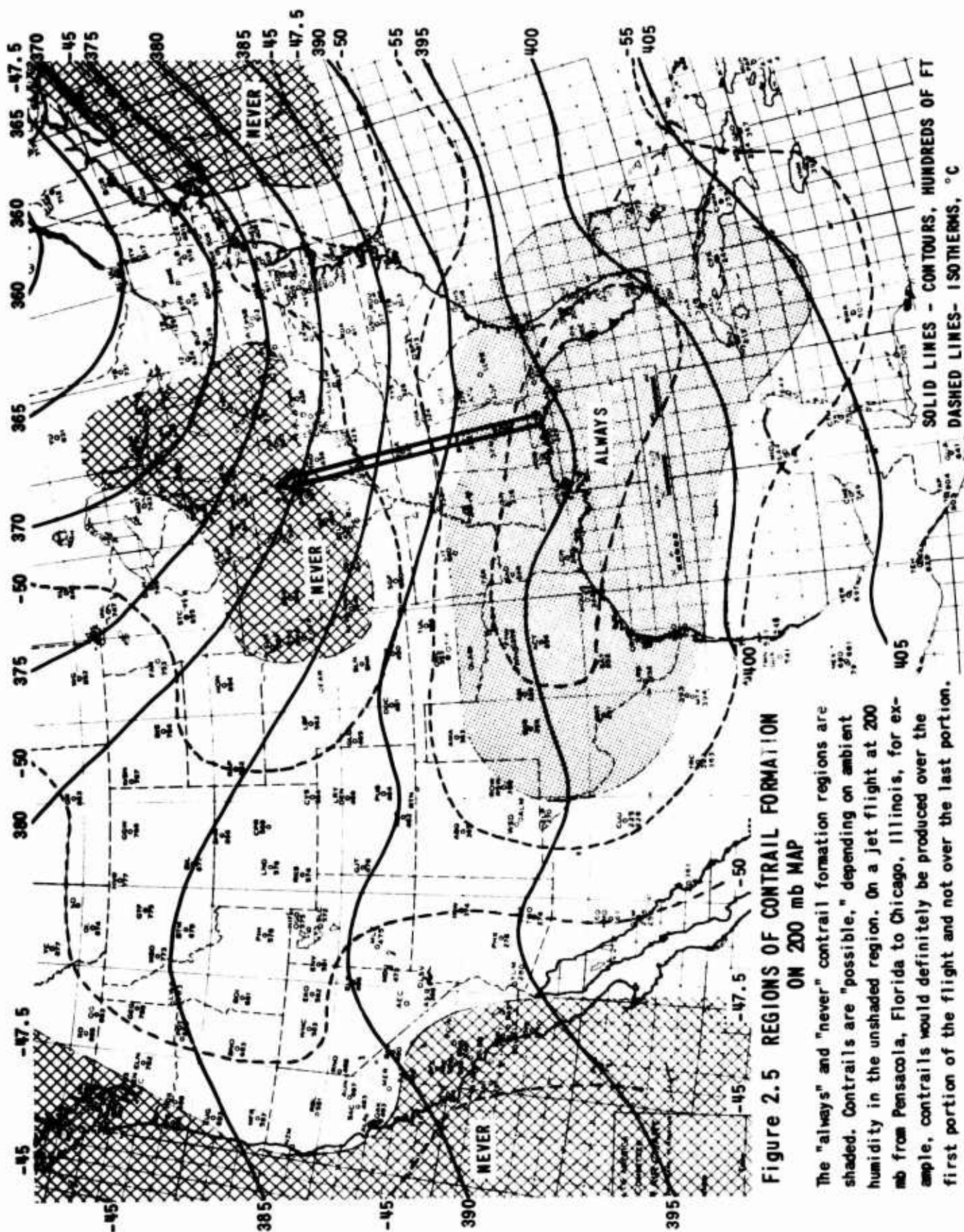
4. Lends itself to contrail persistence forecasts (Section 4).

These charts are intended as a supplement rather than as a replacement to Figure 2.1.

2.6 Contrail Occurrence Regions on Constant Pressure Maps

The prediction of contrails over large areas or on a global basis can effectively be done with standard constant pressure maps. Merely by studying the isotherm patterns already indicated on these maps, the meteorologist can readily identify (and shade appropriately) areas that fulfill the "always," "possible," and "never" contrail occurrence criteria of Figure 2.1 and Table 2.1. For example, on a 200 mb map, areas colder than -56.4°C will always foster contrails; areas warmer than -47.5°C will be free of contrails; and areas with temperatures between -47.5°C and -56.4°C may or may not foster contrails depending on ambient humidity.

Figure 2.5 shows a 200 mb map that has been shaded to depict these areas of contrail expectation. Such a map provides a graphic presentation of the contrail situation, from which the meteorologist and pilot can determine appropriate flight paths. Figure 2.6 illustrates a hemispheric 100 mb map of mean temperature conditions for January 1949 which is useful for long range mission planning.



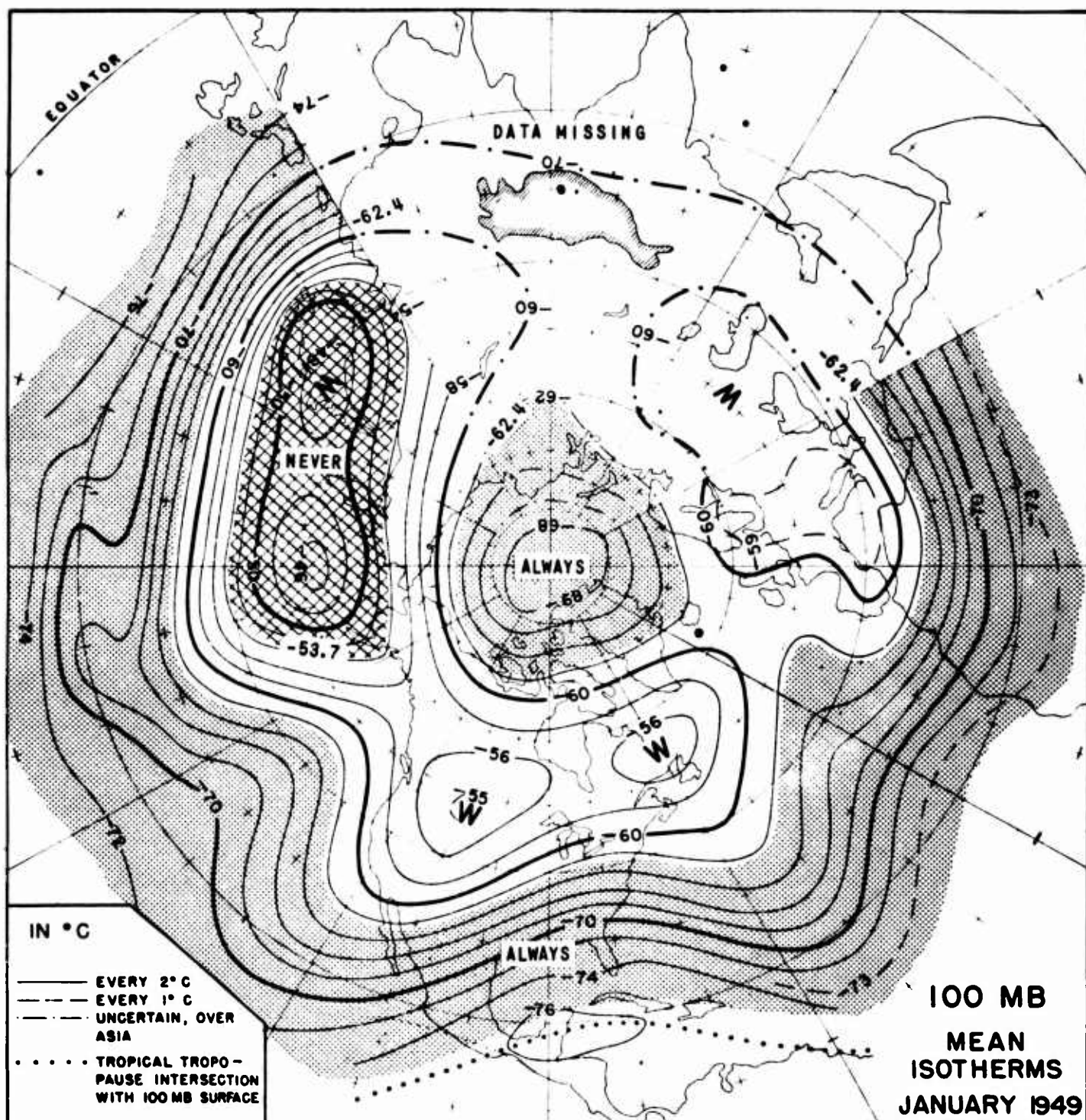


Figure 2.6 MEAN TEMPERATURE AND CONTRAIL FORMATION AT 100 mb [Ref. 7]
 Contrail expectation, is indicated by the shaded ("always" or "never") and unshaded ("possible") regions.

3. VALIDITY OF CONTRAIL PREDICTION METHODS

Contrails are not always observed in regions where theory would dictate that they should form. Principal reasons for this apparent discrepancy are that (1) the visual detection of contrails is dependent on viewing angle with respect to the sun and on the background contrast and (2) contrails do not always freeze.

Laboratory studies [4] have shown that a contrail with 0.004 g/m^3 water concentration is visible only under ideal conditions of observation. A contrail is most distinct when seen at a forward (less than 90°) scattering angle against a contrasting background. Under less favorable viewing conditions - for example, at large scattering angles or with a cirrus background - contrail density must be larger by an order of magnitude or more to be visible. Appendix A describes how the light scattering properties of water and ice clouds decrease as the scattering angle increases. Figure 3.1 illustrates the dependence of contrail detection on the respective positions of the observer, contrail and source of illumination (sun, moon).

Laboratory experiments [4] have provided strong evidence that contrails do not always freeze, especially at relatively warm ambient temperatures close to the T_c point. When contrail droplets do freeze, which is the more common situation, deposition of water vapor onto the ice crystals because of vapor pressure differences is sufficient to fulfill the 0.004 g/m^3 visibility criterion. However, if the contrail droplets do not freeze, contrails can only be produced at ambient temperatures cold enough to provide the necessary amount of observable moisture, i.e. $0.004 - 0.1 \text{ g/m}^3$, depending on viewing conditions. The critical contrail formation temperatures would be shifted to colder values; for the minimum visibility criterion of 0.004 g/m^3 , this shift, as shown in Table 2.1, amounts to $1 - 3^\circ\text{C}$ at typical jet flight altitudes.

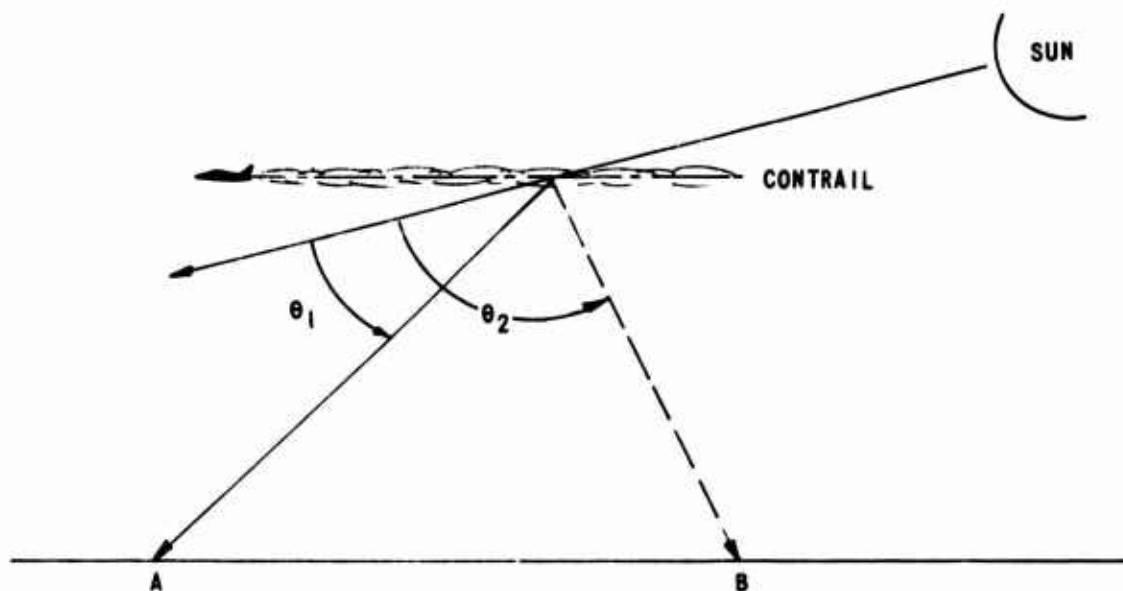


Figure 3.1 CONTRAIL DETECTION AT DIFFERENT SCATTERING ANGLES

Contrails that are just barely visible from the ground at point A, where a forward scattering angle θ is involved, will not be visible at ground point B.

A second implication of liquid contrails is that such trails will quickly evaporate owing to the fact that the higher atmosphere is almost always subsaturated with respect to water.

In summary, either of the previous forecast techniques will indicate the occurrence of contrails at times when (even if p, T and RH data are exactly known) they will not be observed owing to less-than-optimum viewing conditions and the trails not freezing. Thus, the techniques tend to bias contrail forecasts in the affirmative direction.

4. CONTRAIL PERSISTENCE

4.1 Factors Influencing Contrail Persistence

The duration of a contrail depends on ambient relative humidity, final condensate phase, contrail density and atmospheric diffusion characteristics. Quantitative methods for predicting the persistence of contrails have not been prescribed in the past because these influential variables are generally not accurately known at flight altitudes.

Contrails that remain in the liquid phase will evaporate within seconds of the aircraft passage, since the relative humidity at upper levels is practically always less than 100 percent with respect to water and generally less than 90 percent. Contrails that freeze and are subsaturated with respect to ice will usually require a minute or more to disappear depending on the degree of subsaturation, contrail density and environmental mixing. Contrails that freeze in an ice saturated environment will grow and persist for long periods of time; atmospheric wind shear and turbulence will gradually diffuse these ice trails into "cirrus" decks. (Persistent, diffusing contrails will continue to indicate the directional track of the aircraft long after they are formed.)

4.2 Graphical Estimate of Persistence

The mixing ratio-temperature phase diagram can be used to advantage to indicate contrail zones associated with characteristic persistence patterns. Figure 4.1 is a partial reconstruction of Figure 2.3 with an ice saturation curve superimposed. As stated previously, the line tangent to the water saturation curve bounds the range of atmospheric conditions that will foster contrail formation. This region has been divided into three zones which can be interpreted as follows:

1. Zone A - The region occupied by any contrail that forms in this environment must be supersaturated with respect to ice as indicated by the fact that all environmental conditions represented by points in A lie above the ice saturation curve w_i . If the contrail freezes, it will persist. At temperatures colder than T_i , the vertical boundary of Zone A, the contrail region may either be greater or less than ice saturation.

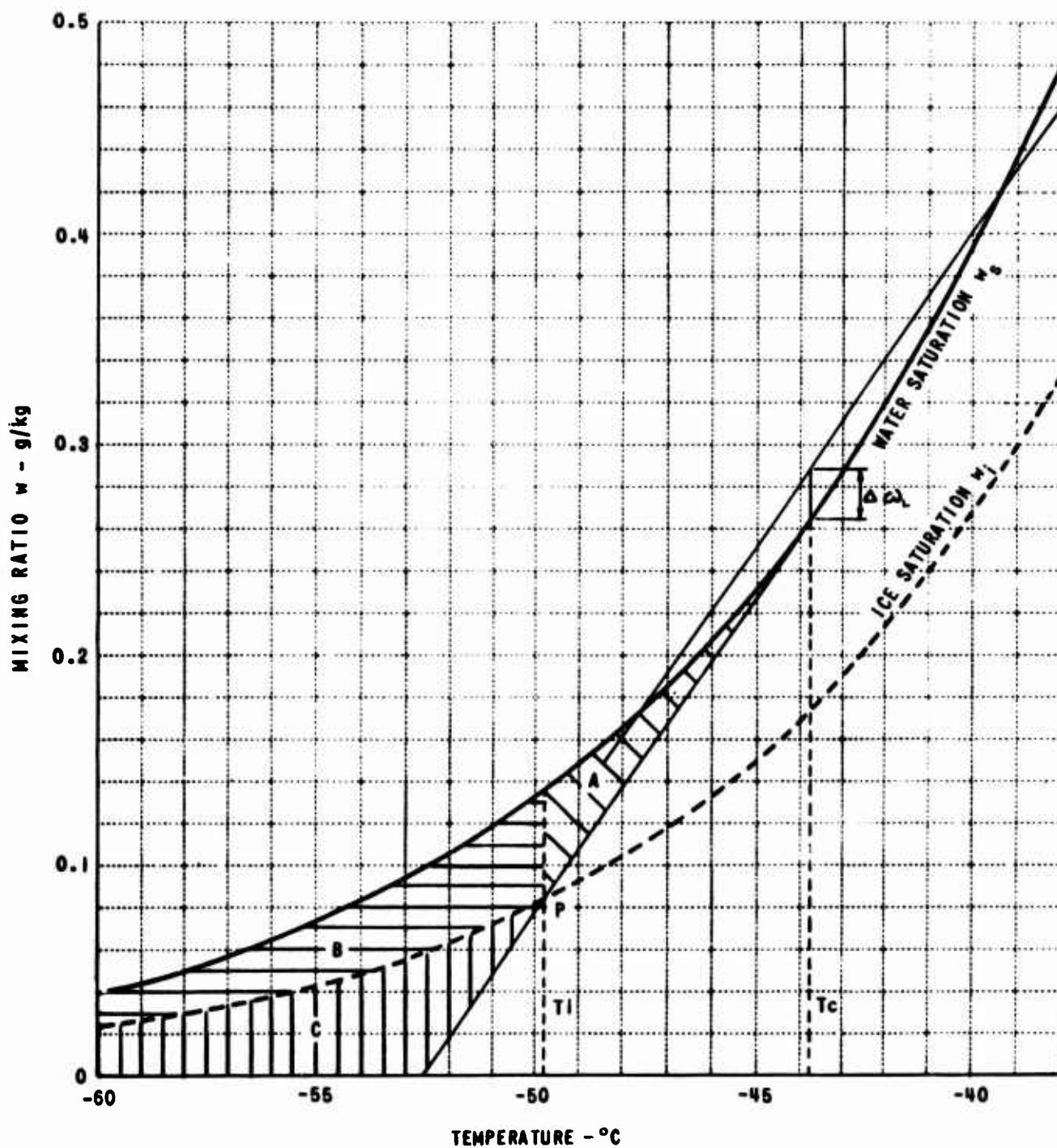


Figure 4.1 CONTRAIL PHASE DIAGRAM - 300 mb

If atmospheric temperature and humidity conditions are characterized by any point in regions A and B, the contrails that form will be super-saturated with respect to ice (exceed w_i curve) and hence persistent. Ambient conditions characterized by region C will result in nonpersistent contrails

2. Zone B - The environment is saturated with respect to ice, and contrails consisting of ice crystals will persist

3. Zone C - The atmosphere is subsaturated with respect to ice, and contrails will not persist. Generally speaking, the colder the temperature within Zone C, the more dense will be the contrail and the longer it will take for the trail to dissipate.

The values of relative humidity with respect to water for ice saturation are given in Table 4.1 as a function of temperature. Each humidity value can be obtained from standard tables [5] or graphically from Figure 4.1, where for a given temperature the value is equal to $100 \frac{w_i}{w_s}$. As shown, ice saturation varies from 75 percent RH at -30°C to 55 percent RH at -60°C . Thus, contrail persistence or nonpersistence can be specified if ambient humidity is known. Though accurate measurements of upper-level humidity are generally not available, inspection of synoptic weather patterns and cloud reports (as discussed in Section 5) will often indicate whether the flight level humidity in question is greater or less than ice saturation.

Contrails which do not freeze will always be nonpersistent. An absolute prediction of nonfreezing contrails cannot be made, although the most probable conditions for such an occurrence can be indicated. In the absence of freezing nuclei, the spontaneous freezing of supercooled droplets is a statistical event depending mainly on temperature and to a lesser extent on drop volume and cooling time [6]. Relatively warm temperatures, small droplets and short cooling times tend to discourage freezing. Referring to Figure 4.1, Zone A usually offers a more favorable region for liquid contrails than do Zones B or C. Not only are temperatures warmer but the amount of excess moisture available for condensation is generally smaller and, hence, droplet growth more limited.

Table 4.1
Relative Humidity with Respect to Water
at Ice Saturation

<u>Temperature</u>	<u>Relative Humidity with Respect to Water</u>
-30°C	74.6 %
-35	71.1
-40	67.8
-45	64.8
-50	61.9
-55	59.4
-60	55.3

Liquid nonpersistent contrails are more apt to occur at lower altitudes where contrail formation at progressively warmer temperatures is possible. Conversely, it is reasonably safe to assume that all contrails will freeze that are formed above 150 mb where minimum formation temperatures T_c are colder than -50°C . (The forecaster should be reminded that even ice trails formed at 150 mb or higher will generally be nonpersistent since the mixing ratio at these altitudes is usually less than that of ice saturation.) Laboratory tests in which miniature contrails were produced in an altitude chamber have shown the occasional existence of nonpersistent liquid droplets at temperatures down to approximately -50°C ; however, at temperatures colder than -50°C , ice trails were observed in every case [4].

4.3 Forecast Rules

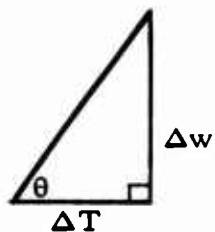
The effect of fuel exhaust products on droplet purity and freezing is not known, nor are extensive data available on the concentration of freezing nuclei within the contrail wake at flight altitude. Because of these uncertainties and because spontaneous freezing is a probabilistic event, it is best to adopt the following conservative criteria in order to forecast contrail persistence:

1. Assume that contrails always freeze (quite certainly the case above 150 mb); use a phase diagram like Figure 4.1 to determine super- or subsaturation relative to ice. A persistent or nonpersistent forecast would be made accordingly.
2. Qualify a nonpersistent forecast by a measure of the concentration of contrail moisture $\Delta\omega_L$. Other conditions being equal, contrail dissipation time will increase as $\Delta\omega_L$ increases. The numerical relationship between $\Delta\omega_L$ and contrail decay time awaits the accumulation of well-monitored flight data.
3. Recognize that contrails that do not freeze will be nonpersistent and will degrade a persistence forecast based on expected ice saturation conditions. This possibility increases at contrail formation temperatures within about 10°C on the cold side of T_c at altitudes below 45,000 feet (pressures greater than 150 mb).

4.4 Graphical Aids

For field use it is recommended that a separate phase diagram similar to Figure 4.1, Contrail Phase Diagram - 300 mb, be constructed for each pressure level of interest; charts for each 50 mb level between 300 and 150 mb will cover the customary flight altitudes of conventional jet aircraft. Values of saturation mixing ratio over water and over ice versus pressure and temperature are contained in Appendix B. A transparent plastic overlay for sketching $\Delta w/\Delta T$ lines with an appropriate right triangle, as illustrated in Figure 4.2, will add to a chart's utility.

Figure 4.2 Plotting Triangle - $\Delta w/\Delta T$ Slope



$$\theta = \tan^{-1} \frac{\Delta w}{\Delta T} = \tan^{-1} 0.0295 \text{ g/kg}^{\circ}\text{C}$$

These charts can be used both for predicting contrail formation as previously discussed in Section 2.5 and for estimating contrail persistence.

5. CONTRAIL CLIMATOLOGY

5.1 General Considerations

The forecasting of condensation trails can be accomplished with high accuracy when ambient pressure, temperature and relative humidity are known. If any of these parameters are in doubt, as is generally the case over a given flight path, prevailing macroscale weather patterns can often provide supplementary data for making more accurate contrail forecasts. Geographic and altitudinal considerations can further refine a forecast. Relationships between condensation trail occurrence and these climatological features can also assist in long range military planning.

Attempts to correlate contrail occurrences with weather patterns on the surface map would be unreliable because of complex relationships between surface and upper-level weather patterns. The following distinctive features of upper-level weather maps can be linked with characteristic temperature and humidity patterns and hence with contrail occurrence:

1. upper-level pressure cells
2. jet streams
3. the tropopause
4. regions of high cloud cover.

Critical contrail formation temperatures for saturated and dry environments, extracted from Table 2.1, appear below for ready reference.

	T_c <u>Saturated Environment</u>	T_d <u>Dry Environment</u>
100 mb	-53.7°C	-62.4°C
200	-47.5	-56.4
300	-43.7	-52.6

By comparing these critical values with the characteristic temperature - moisture patterns of the meteorological features outlined above, it is possible to predict whether contrails are likely to be associated with the weather features. As mentioned earlier, for example, a weather pattern characterized by temperatures warmer than -47.5°C at 200 mb clearly defines a region where contrails will not occur. Similarly, a weather pattern associated with temperatures colder than -56.4°C at the same pressure level would establish a region of contrail occurrence. Between these two temperatures, corresponding to saturated and dry atmospheric conditions respectively, success in prediction of contrail occurrence depends once again on knowledge of relative humidity. A qualitative classification of macroscale meteorological patterns according to typical moisture content can provide additional relative humidity information and strengthen the confidence level of contrail forecasts.

The subsequent analysis considers primarily levels between 25,000 ft and 55,000 ft, i.e., approximately 300-100 mb.

5.2 Upper-Level Pressure Systems

Upper-level tropospheric troughs (or lows) are relatively cold and ridges (or highs) relatively warm. Just the opposite is true in the lower stratosphere (below 80,000 ft), troughs being warm and ridges cold. The temperature difference between adjacent pressure cells is appreciable, $10-15^{\circ}\text{C}$ not being an extreme differential. Therefore, in the lower stratosphere, a pressure ridge is more apt to foster contrails than a trough, whereas the reverse is true in the upper troposphere.

An analysis of mean global temperature patterns on a seasonal basis can be made from appropriate upper-level charts [7]. From average temperature values over the northern hemisphere at 300, 200, and 100 mb the following regions generally unfavorable for contrail formation can be delineated:

1. In the vicinity of low-pressure cells in the lower stratosphere and near high-pressure cells in the upper troposphere.

2. Latitudes south of 35°N in winter and south of 60°N in summer at 300 mb.
3. Latitudes north of 60°N in summer at 200 mb.
4. North of 55°N in summer and between 40 and 60°N in winter at 100 mb.

In effect, the above areas approximate the warmest regions at a given pressure level. By similarly delineating the coldest regions, areas most conducive to contrail formation can be prescribed. Figure 2.6, Mean Temperatures and Contrail Expectation at 100 mb, illustrates this procedure. Since the thermal forecast aids presented are based on average temperature conditions, a meteorologist must exercise discretion in applying such rules when atypical conditions are met.

5.3 The Jet Stream

A number of investigators have noted the higher incidence of clouds and increased moisture on the right-hand side of jet streams looking downwind (high pressure side). It is, therefore, tempting to conclude that aircraft hoping to avoid contrails should select flight paths on the left side of a jet stream. This conclusion is generally correct for stratospheric flights, but it is invalid for levels below the tropopause where, as shown in Figure 5.1, the left-hand side of the jet is considerably colder than the right. The lower temperatures often give rise to conditions suitable for contrail formation, despite the dryer environment.

At the level of the jet stream, the average relative humidity with respect to ice is about 50 percent 250-300 miles from the jet stream axis on the high-pressure side, and only about 10 percent at the same distance on the low-pressure side [8]. Though considerable variation in humidity is noted, cloud cover is very rare on the low-pressure side at distances exceeding 100 miles. The jet stream core itself covers a wide humidity range, dry cores being associated with relatively warm jets and moist cores with relatively cold jets.

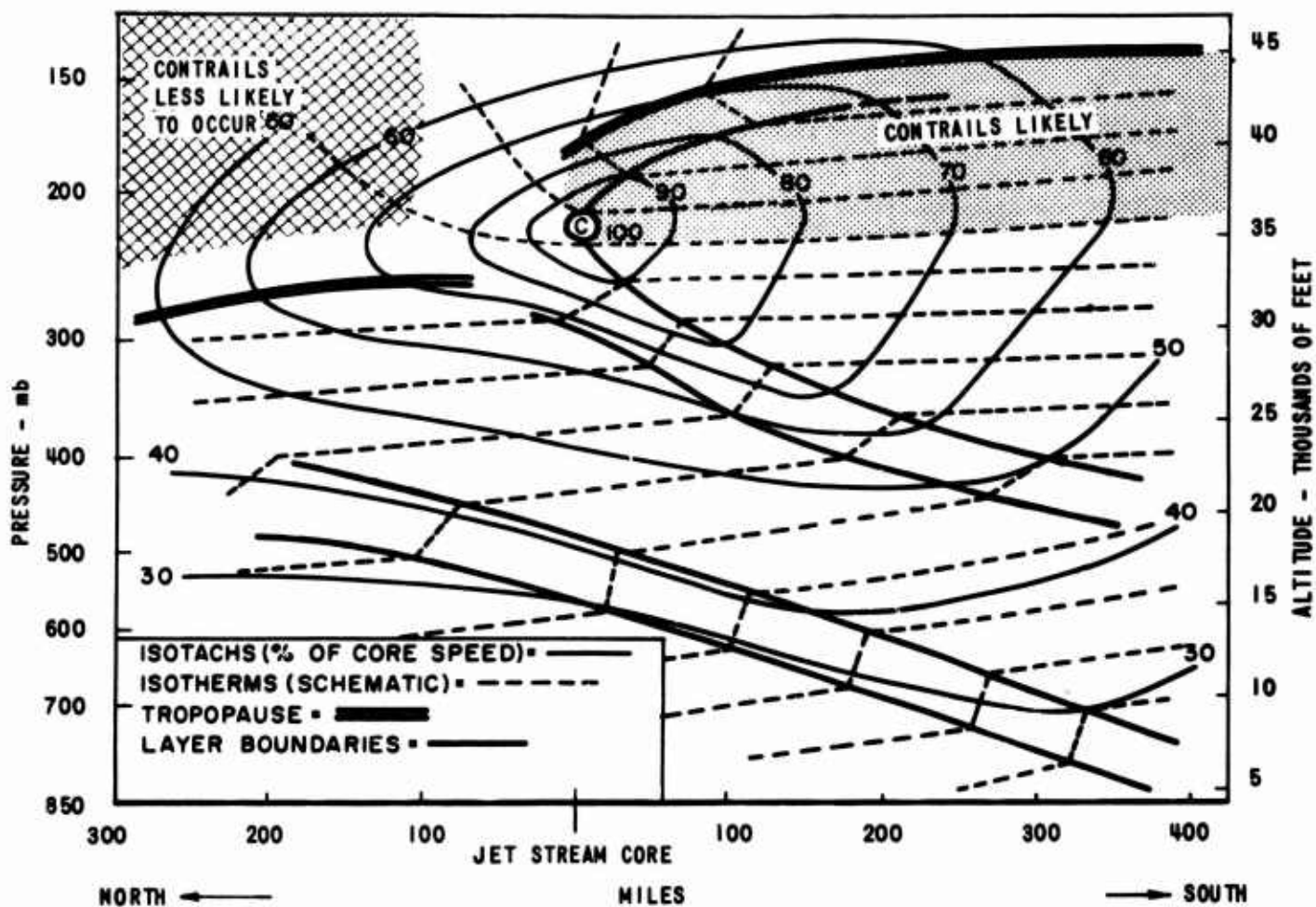
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It can be assumed that aircraft flown in the stratosphere on the left side of jet streams (looking downstream) at distances of 100-300 miles from the jet axis stand a good chance of not forming contrails. This "safe" zone may be expanded to include the jet core itself, in the case of relatively warm jet streams.

5.4 The Tropopause

The tropopause varies in height from a low at the pole (20-30,000 feet) to a maximum height at the equator (50-60,000 feet) as shown in Figure 5.2. On any given vertical cross section, the coldest layer of air is usually found at or near the tropopause. The tropopause also tends to cap the upward flux of water vapor so that relatively high concentrations of moisture often exist just below the tropopause base. This combination of low temperature and high relative humidity makes the immediate tropopause region generally favorable for contrail formation.

Humidity measurements aloft have indicated that at altitudes commencing 5000 to 10,000 feet above the tropopause, the relative humidity with respect to ice is generally less than 10 percent and often less than 1 percent [10]. Thus, at 100 mb and sometimes at 200 mb, depending on tropopause height, a dry environment can be safely assumed; contrails will form only if the temperature is colder than the T_d values in Table 2.1.

Mean temperature charts show that at the intersection of the 200 mb surface with the tropopause, the average temperature varies between about -53°C and -56°C . As a result, the intersection region (approximately 30°N in winter and 45°N in summer) is very conducive to contrail formation. Consequently, aircraft hoping to elude detection from ground observers should routinely avoid the tropopause level.

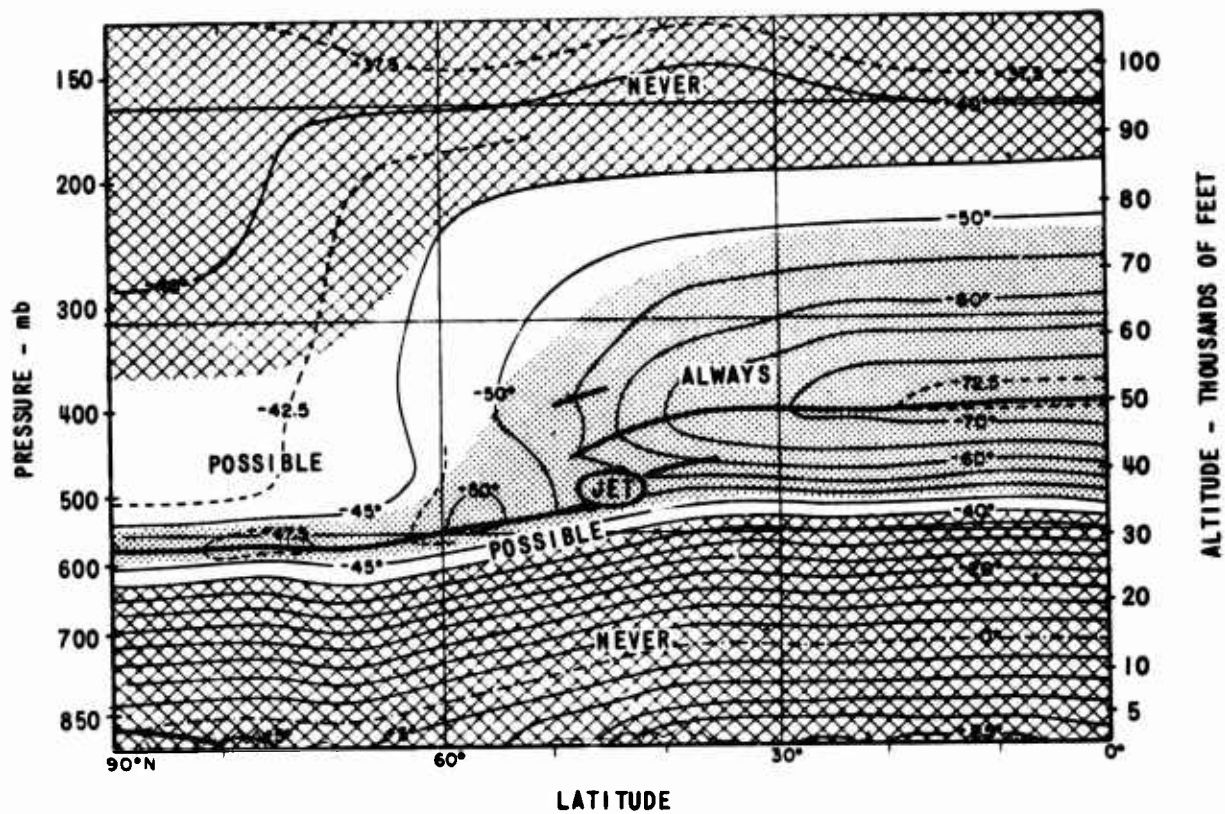


Figure 5.2 TROPOPAUSE STRUCTURE - JULY [Ref. 1]

Cold temperatures in the vicinity of the tropopause (heavy line) are conducive to contrail formation.

5.5 Cirrus Cloud Patterns

It is fairly obvious that extensive cloud cover at any level is indicative of a relatively moist environment; in addition, the changing quality of clouds provides information about ambient relative humidity. Where cirrus clouds are present, but are neither growing or dissipating appreciably, a relative humidity of 100 percent with respect to ice can be assumed at cloud level. This state corresponds to a relative humidity with respect to water of about 60 percent at -55°C . If the cirrus clouds are thickening and growing, atmospheric conditions must be supersaturated with respect to ice so that a relative humidity of 70-90 percent with respect to water might be more representative. Similarly, dissipating cirrus decks at -55°C would indicate an ambient relative humidity less than 60 percent. Appropriate relative humidity values at other temperatures are indicated in Table 4.1. This type of information, when coupled with air temperature information from radiosondes or pilot reports, can improve a contrail forecast based on ambient temperature alone.

5.6 Altitudes Above 60,000 ft

The saturation mixing ratio of air at constant temperature increases almost linearly with decreasing pressure. Hence, progressing upward in the stratosphere where temperatures are approximately isothermal and in the lower mesosphere (up to 100,000 ft) where temperature tends to increase slightly, the amount of water required to saturate a given mass of air steadily increases. For this reason the probability of forming contrails above 60,000 ft steadily decreases.

For all practical purposes it can be assumed that aircraft contrails do not form above 75,000 ft. Appendix C lists the probability of forming contrails as a function of altitude and latitude for North America [12].

5.7 Northern Hemisphere Climatological Forecast Rules - Summary

A. Good Flight Paths - Generally Unfavorable for Contrail Formation

1. High-pressure areas in the upper troposphere and low-pressure areas in the lower stratosphere (below 80,000 ft).
2. Latitudes north of 55°N in the stratosphere (200-100 mb) during the summer; mid-latitudes between 40°N and 60°N at 100 mb during the winter season.
3. South of 35°N in winter and south of 60°N in summer at 300 mb (upper troposphere).
4. On the left side of jet streams looking downstream, 100-300 miles from the jet axis. For "warm" jet streams, the core itself may be included.
5. At altitudes 10,000 feet or more above the tropopause, temperature criteria for the appropriate level permitting. Dry environment T_d values should be assumed.
6. Areas where cirrus clouds are absent, temperature criteria permitting.
7. Altitudes above 75,000 feet.

B. Flight Paths to Avoid - Generally Favorable for Contrail Formation

1. Low-pressure areas in the upper troposphere and high-pressure areas in the lower stratosphere.
2. Entire 200 mb level in winter. At 100 mb, south of 45°N in summer; south of 40°N and north of 60°N in winter.
3. North of 35°N in winter and north of 60°N in summer at 300 mb.
4. On the right side of jet streams looking downstream, up to about 400 miles from the axis.
5. The tropopause level plus or minus about 2000 feet.
6. Areas where cirrus clouds are present. (If the clouds are sufficiently dense, they may obscure contrails and thus be beneficial.)

6. SUMMARY

Results of theoretical and experimental studies regarding the prediction, formation and detection of aircraft condensation trails are summarized as follows:

1. Laboratory and field experiments show that Appleman's well known graphical method (Fig. 2.1) for predicting contrail occurrence can be used with confidence by the meteorologist, with the following qualifications:

a) The $\Delta w/\Delta T$ ratio that expresses the decrease in moisture and temperature in the aircraft exhaust wake is best represented by a value of $0.0295 \text{ g/kg}^\circ\text{C}$.

b) The criterion for a visible contrail, namely one possessing a water concentration of 0.004 g/m^3 , is valid only for ideal conditions of observation. Optimum viewing conditions involve a forward scattering angle of the sun's rays and a contrasting sky background. Under less favorable viewing conditions, a contrail may require upwards of 0.1 g/m^3 of condensate to be visible.

c) There is strong evidence that the final phase of contrails is not always ice. Contrails that remain in the liquid phase will require colder temperatures ($2\text{-}3^\circ\text{C}$) to satisfy the minimum visibility criterion (0.004 g/m^3), and will always be nonpersistent.

2. An alternate graphical method of contrail prediction yielding equivalent results is presented. This graphical presentation is particularly useful for (a) contrail persistence forecasts, (b) expressing contrail density and (c) illustrating the cloud physics principles involved in contrail formation.

3. Standard constant-pressure maps with isotherm patterns included can be effectively used to delineate contrail expectation over large areas and along specific flight paths.

4. Contrail persistence is a function of ambient humidity, final condensate phase, contrail density and atmospheric diffusion characteristics. Water contrails will practically always evaporate within a matter of seconds. Contrails consisting of ice particles, the more common situation, will persist for hours if environmental conditions exceed ice saturation, i.e., exceed ambient relative humidities of approximately 60-70 percent. When the ambient humidity is less than ice saturation, contrails comprised of ice crystals will sublime in seconds to minutes depending on contrail density.

5. Certain recognizable weather patterns offer assistance in the forecasting of contrails where accurate knowledge of temperature and humidity at the pressure level of interest is not complete. Consideration of certain synoptic patterns and climatology offers supplementary information to augment standard prediction techniques. Such aids are also helpful for advance planning of long-range military operations.

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APPENDIX A

CONTRAIL VISIBILITY

A condensation trail becomes visible to an observer if the luminance contrast exceeds a certain threshold value. The contrast is expressed as

$$C = \frac{B_T - B_b}{B_b},$$

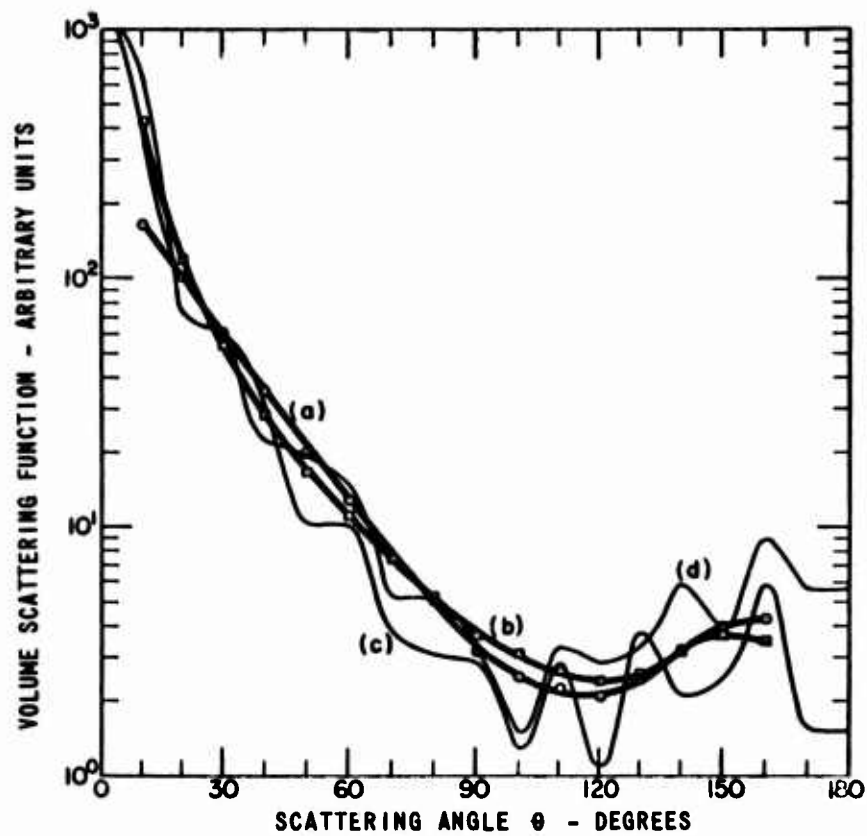
where C is the fractional luminance difference between the trail and the background, B_T is the luminance of the trail and B_b is the luminance of the background. The luminance of a light source, or of an illuminated object, is the quantity of luminous flux emitted per unit solid angle by each unit of source area perpendicular to the direction of observation.

The contrail is luminous by virtue of the light scattered by trail constituents in the direction of the observer. Therefore, the luminance of the trail depends on both the light-scattering properties of the particles in the trail and the intensity and direction of the incident light. The scattering from a particle depends on its size and shape and the scattering from a small volume of the trail depends on the concentration and the distribution in size and shape of the particles.

In a typical contrail the droplets and/or ice crystals are separated by such distances that light scattered by any trail constituent generally proceeds undisturbed through the remainder of the trail. Therefore, the contribution of scattering by the condensate to trail luminance in the direction of the observer is equal to the sum of the light flux per unit solid angle scattered by all particles through a unit area of trail surface facing the observer. This quantity of light is equal to the product of the intensity of illumination and the differential scattering cross section of the particles at the given scattering angle. The scattering angle in this case is the angle subtended at the contrail by the observer and the sun's rays.

The volumetric differential scattering cross section is strongly dependent upon the scattering angle as shown in Figure A-1. It is apparent that the scattering function decreases by over two orders of magnitude as the scattering angle increases from 0 — 180 degrees. Thus the visual detection of contrails is highly dependent on the angle of observation.

Referring back in the text to Figure 3.1, Contrail Detection at Different Scattering Angles, imagine two observers at points A and B attempting to observe possible contrails aloft. If the observer at A is just able to observe the contrail at the forward scattering angle $\theta_1 = 30^\circ$, then the observer at B, presented with a back scattering angle $\theta_2 = 100^\circ$, will receive less than 1/50 of the light received at A. Hence the observer at B will not be able to see the contrail.



**Figure A-1 NORMALIZED VOLUMETRIC DIFFERENTIAL SCATTERING
CROSS SECTION**

- (a) Measured in water cloud [4]
- (b) Measured in ice and cloud [4]
- (c), (d) Calculated for monodispersed
clouds of 1.4 and 1.7 μ diameter
droplets respectively [13]

APPENDIX B

Saturation Mixing Ratio vs. Temperature and Pressure With
Respect to Water w_s and With Respect to Ice w_i

Temp. °C	300 mb				250 mb				200 mb				150 mb				100 mb				50 mb			
	w_s	w_i	$\frac{w_s}{w_i}$	$\frac{w_i}{w_s}$	w_s	w_i	$\frac{w_s}{w_i}$	$\frac{w_i}{w_s}$	w_s	w_i	$\frac{w_s}{w_i}$	$\frac{w_i}{w_s}$	w_s	w_i	$\frac{w_s}{w_i}$	$\frac{w_i}{w_s}$	w_s	w_i	$\frac{w_s}{w_i}$	$\frac{w_i}{w_s}$	w_s	w_i	$\frac{w_s}{w_i}$	$\frac{w_i}{w_s}$
-30°C	1.058	0.710	1.270	0.948	1.270	0.948	1.588	1.185	2.119	1.580	3.183	2.373	6.366	4.746										
-35	0.653	0.464	0.783	0.557	0.783	0.557	0.979	0.696	1.305	0.928	1.960	1.392	3.920	2.784										
-40	0.393	0.267	0.472	0.320	0.472	0.320	0.589	0.400	0.786	0.533	1.179	0.800	2.358	1.599										
-45	0.231	0.150	0.277	0.180	0.277	0.180	0.346	0.224	0.461	0.300	0.692	0.449	1.384	0.898										
-50	0.132	0.082	0.158	0.098	0.158	0.098	0.198	0.123	0.264	0.163	0.396	0.245	0.792	0.490										
-55	0.073	0.044	0.088	0.052	0.088	0.052	0.110	0.065	0.147	0.087	0.220	0.131	0.440	0.261										
-60	0.040	0.022	0.048	0.027	0.048	0.027	0.060	0.034	0.080	0.045	0.120	0.067	0.241	0.135										
-65	0.020	0.011	0.025	0.013	0.025	0.013	0.031	0.017	0.041	0.022	0.061	0.034	0.123	0.067										
-70	0.010	0.005	0.123	0.007	0.123	0.007	0.015	0.008	0.020	0.019	0.031	0.016	0.061	0.033										

g/kg

APPENDIX C
Contrail Formation Probability*

%
January and July - North America [10]

		LATITUDE (°N)													
		January						July							
ALTITUDE (thousands of feet)		20°N		30		40		50		60		70		80	
		20°N	30	40	50	60	70	80	20°N	30	40	50	60	70	80
10		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	1	8	16	22							
20		0	0	0	5	16	30	50							
		0	0	0	34	70	96	99							
30		0	0	16	59	74	94	96							
		0	16	84	96	99	100	100							
40		27	50	59	44	44	50	64							
		97	98	97	93	88	90	95							
50		99	88	50	17	20	29	30							
		100	100	98	74	70	71	80							
60		99	74	8	3	12	20	28							
		100	100	84	42	50	73	80							
70		10	3	0	0	1	7	24							
		84	65	16	3	23	60	84							
80		0	0	0	0	0	3	5							
		0	0	0	0	3	33	65							
90		0	0	0	0	0	0	4							
		0	0	0	0	0	12	40							
100		0	0	0	0	0	0	1							
		0	0	0	0	0	3	23							

* Figures in the upper line for each altitude are for 30% ambient relative humidity. Those in the lower line are for 100% ambient relative humidity.